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Innovation Diffusion in Time and Space: Effects of Social Information and of Income Inequality

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Abstract

In this paper we consider the spread of modern technological innovations. We contrast social learning and threshold heterogeneity models of innovation diffusion, and show how the typical temporal evolution of the distribution of adopters may be consistent with either explanation. Noting the likelihood that each model contains some useful independent explanatory power, we introduce a combined model. We also consider a spatially-structured population in which the spread of an innovation by social influence is modelled as a reaction-diffusion system, and show that the typical spatiotemporal evolution of the distribution is also consistent with a heterogeneity explanation. Additional contextual information is required to estimate the relative importance of social learning and of economic inequalities in observed adoption lags.

Keywords: innovation diffusion, social learning, threshold heterogeneity, income inequality, Gini coefficient, gamma distribution, reaction-diffusion, travelling wave.

1. Introduction

Reaction-diffusion systems based on the Fisher-KPP equation

$$\frac{\partial Y}{\partial t}(t) = D\Delta Y(t) + \alpha(1 - Y(t)) \tag{1}$$

have been studied in various social science contexts. One major area of application has been the modeling of front propagation during major episodes of population dispersal (for instance, the spread of prehistoric hunter-gatherers into the Americas at the end of the last Ice Age, or the spread of prehistoric farming in Europe; [24]). Reaction-diffusion systems are also widely used to model epidemic processes of disease spread. Other recent applications focus on contact-mediated social processes where cultural attributes as-it-were compete for their human hosts (for example, language competition models where one language loses its speakers and goes extinct; [17]).

In this paper we will study the spread of modern technological innovations using a reaction-diffusion framework. The premise of such a framework is that rates of uptake of an innovation are dependent on social influence. The reaction term, which determines local rates of increase in the numbers of adopters, assumes some degree of dependence on local numbers of prior adopters. The diffusion term assumes some additional degree of dependence on contact and interaction with other adopters in one's more extended neighbourhood or social network.

Such models might not seem the obvious choice for a marketing analysis of a new product. Producers routinely try to overcome time delays in local product take-off using advertising, which is designed to accelerate take-off by increasing consumer awareness of the new product - thereby overwhelming any local frequency-dependent copying biases. Producers also try to overcome any delay in market penetration due to consumers' dependency on what is going on in their own neighbourhoods, by simultaneous distribution of the new product at numerous widely-dispersed centres of local diffusion. The reasons why producers of new products should oppose a purely local contact-driven marketing approach are obvious. When such dynamics are dominant, their market will grow much more slowly. If local rate of growth in the population of adopters depends on the prior numbers of adopters then take-off is delayed, which incurs costs to the producer and increases the risk of invasion of the developing market by competitors. If the spatial spread of the innovation is based on local contact processes radiating out from a single diffusion pole, then global saturation of a mature market is greatly delayed and again there is an increased risk of invasion of the developing market by competitors starting to distribute their own products from other spatial locations.

In light of these considerations, it may seem surprising that empirical new product adoption curves often have a temporal pattern consistent with a strong frequency-dependent social bias on purchasers' decisions. In addition, in some well-studied cases of agricultural innovations, there is a spatial pattern consistent with locally-biased diffusive spread. In this paper we will outline a reaction-diffusion framework for modelling such patterns as the outcome of contact-biased adoption decisions, and we will also outline an alternative framework which assumes that adoption delays are due to underlying adopter heterogeneity (which may be spatially structured). We shall conclude by asking which model makes more realistic assumptions about the underlying microscopic behaviour of individual adopters, and by showing that the two models are not mutually exclusive: a combined model can be proposed that incorporates both sets of effects.

2. The Reaction-Diffusion Framework

2.1. The reaction term

A social influence-oriented model of innovation diffusion is provided by dual inheritance theory. In this approach, it is assumed that the majority of human behaviour is acquired through social learning ([3], [4], [15]). Boyd and Richerson ([3]) distinguish several different decision mechanisms affecting the adoption of new cultural traits:

- Guided variation - the selective retention of variants (including novel variants) found to be efficacious by our own experiments,

- Direct biased transmission - the selective copying of variants that we can see have been tested and found efficacious by others,
- Indirect biased transmission - the selective copying of variants from individuals who possess qualities and attributes to which we aspire in ourselves, but without any direct evidence of the efficacy of the copied variant,
- Conformist or anti-conformist transmission - the selective copying of variants from individuals on the basis of their commonness or rarity, but without direct evidence of the efficacy of the copied variant.

The temporal dynamic of the spread of a cultural variant through social learning can be modelled by the following differential equation

$$\frac{\partial Y(t)}{\partial t} = \xi(P - PY(t)) + \eta \mathrm{B}(1 - \mathrm{Y(t)})\mathrm{Y(t)} \qquad (2)$$

where *Y(t)* describes the proportion of the population which has already adopted the variant at time *t*. The constants ξ and η represent the fractions of individuals in population that rely respectively on guided variation and on biased transmission. The parameter *P* is the probability of the innovation being adopted as a result of the adopter's own testing of its efficacy, and the parameter *B* represents the effects of selective copying on the spread of the variant. In this context we have to distinguish between the cases where *B* is a constant and where *B* depends on the frequency *Y* itself. In the first case equation (2) includes the effects of direct and indirect transmission only, and the proportion of the population which has adopted at time *t* can be expressed explicitly by

$$Y(t) = \frac{1 - \mathrm{e}^{-(\xi \mathrm{P}_1 + \eta \mathrm{B})t}}{\frac{\eta B}{\xi P_1} \mathrm{e}^{-(\xi \mathrm{P}_1 + \eta \mathrm{B})t} + 1}. \qquad (3)$$

On the other hand conformist-biased transmission - where the rate has a positive frequency-dependence - can be included by setting

$$B = B(Y) = b(1 - a) + a(Y(t) - c_B)$$

([3]). The constant component *b(1-a)* models the influence of direct and indirect biased transmission, whereas $a(Y(t)-c_B)$ describes the influence of conformist-biased transmission. The parameter *a* is a measure of the strength of that conformist bias. As mentioned in ([15]) *a* should be chosen to be rather small, otherwise conformist bias makes the spread of an initially rare variant impossible. The constant c_B defines a 'commonness threshold'. Only variants with frequencies above this threshold are supported by conformist bias.

To illustrate the general spread dynamic given by equation (2), Figure 1 shows cumulative adoption curves for different parameter constellations. The influence of the individual learning component is assumed to be weak and the proportions ξ and η are chosen to be 0.5. We can derive that a variant is diffused successfully by all the different copying strategies if $B>0$. But in contrast if $B<0$ the innovation is not promoted by these copying strategies and will only increase its frequency by independent learning (compare the cumulative adoption curve for the parameters a=0.2, b=0.1 in Figure 1). If we introduce a conformist bias to the copying strategy then a comparison of the cumulative adoption curves with the parameters a=0, b=0.15 and a=0.07, b=0.15 which obviously differ only in the respective absence (a=0) and presence (a=0.07) of the conformist biased component shows that such frequency dependent biases are able to produce long tails (a more delayed take-off) at the beginning. In contrast the parameter b influences the steepness of the curve. The bigger this coefficient (i.e. the stronger the effect of the other copying biases on adoption rates), the steeper the cumulative adoption curve - and the faster the variant spreads through the whole population.

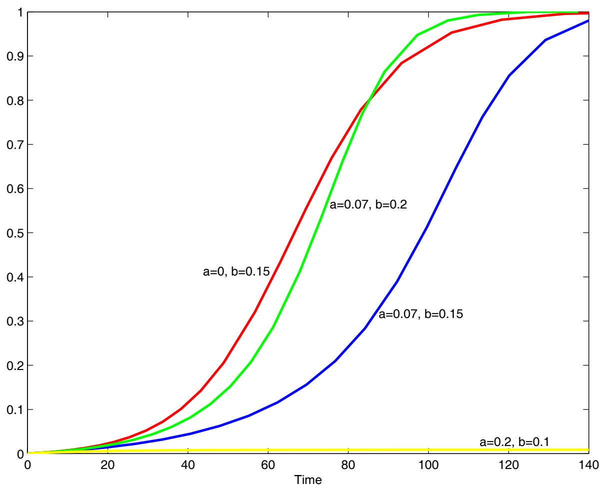


Fig. 1: Cumulative adoption curves for different parameter constellations.

In marketing science, a similar discussion has taken place with a much longer history. Bass' influential model ([1]) proposes that the population of adopters can be divided into independent adopters ('innovators') and imitators, and that the shape of the cumulative adoption curve will vary as a function of their relative importance. The Bass Model includes an innovation coefficient, p, representing the fraction of the population who will adopt the innovation regardless of the number of prior adopters, and an imitation coefficient, q, representing the fraction of the population whose choice is influenced by the number of previous adopters. The basic model states that

$$P(t) = p + qY(t)$$

where P is the probability of adoption by those who have not yet adopted at time t, and $Y(t)$ is the frequency of existing adopters at time t. This can then be expressed as a population rate of increase [25]

$$\frac{\partial Y}{\partial t}(t) = (p + qY(t))(1 - Y(t)) \,. \tag{4}$$

In cases where $q>p$, adoption will increase to reach an internal peak before declining, leading to an S-shaped cumulative adoption curve. In cases where $q \leq p$, adoption rates will be at their maximum initially and then tail off, leading to an r-shaped cumulative adoption curve. The empirical ratio q/p gives an index of the relative importance of innovativeness and of imitation in the diffusion of a particular new cultural trait, and is a shape parameter for the cumulative adoption curve. Empirically, the parameters p and q can be estimated from discrete time series data by regression techniques [1].

A comparison of both approaches shows that without considering conformist bias, the cumulative adoption curves for the spread of innovations coincide exactly. By setting $\tilde{p} = \xi P$ and $\tilde{q} = \eta B$ it is obvious that equations (2) and (4) are the same.

2.2 The diffusion term

A pioneer in the field of spatial diffusion was the Swedish geographer Torsten Hägerstrand. His main contribution was the concept of diffusion as a predictable space-time process and the introduction of Monte-Carlo simulation techniques in this field. He hypothesised that for an innovation to diffuse over time and space, there must be a mechanism of contact and persuasion to transmit the phenomenon ([21]). He assumed that social contact is localised and that diffusion is determined by the dimensions of the potential adopters' 'mean information field' ([13]). Hägerstrand's Monte Carlo simulation model (which is discrete in time and space) implements this contact structure: in each time period every adopter makes contact with other persons (the number depends on the network structure) with a likelihood based on these acquaintance fields. Since they are not known an average field is used to describe the probability of contact at different distances and directions. Furthermore, this field can be affected by geographical barriers. Potential adopters also differ in the number of contacts they needed to make with existing adopters before they adopt the innovation themselves ([21]). Hägerstrand's model was applied to explain agricultural innovation diffusion processes in his native Sweden.

Hägerstrand's model was very influential in human geography in the 1960s and 1970s, and led to further developments in modelling that re-created a Fisher-type reaction-diffusion model without explicit awareness of the parallels (e.g. [22]). In a reaction-diffusion framework, we would represent the contact field isotropically by using it to scale the diffusion constant D (or by using an integral formulation that permits varying functions describing the effect of distance on contact frequency), yielding a system for the propagation of innovation adoption waves:

$$\frac{\partial Y}{\partial t}(t,x) = D\Delta Y + \xi(P - PY) + \eta B(1-Y)Y \ . \qquad (5)$$

The diffusion part $D\Delta Y$ describes the spread of the innovation in space. The parameter D can be interpreted as a measure of the mean information field or in other words the spatial scale of the population's social interaction network.

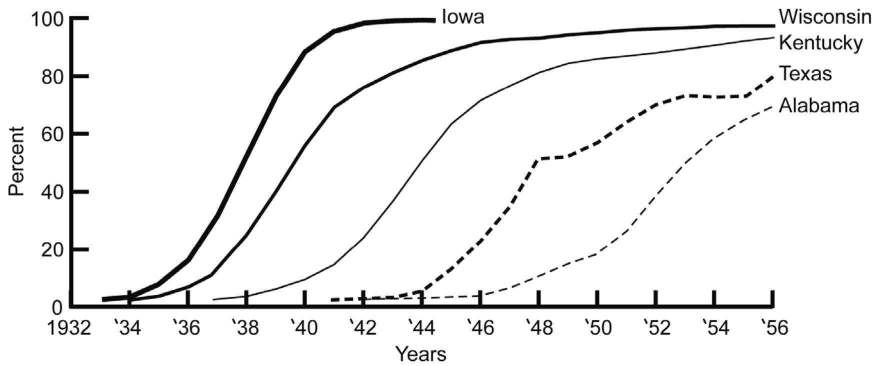


Fig. 2: Percent of total corn acreage planted with the hybrid strain, by state (redrawn after [10], Fig. 1).

The use of such models was encouraged by empirical case studies. When several regional adoption curves for the same cultural trait are plotted side-by-side, they can reveal a spatial lag in adoption consistent with a contagion-diffusion process. Figure 2 illustrates this, showing the adoption curves for hybrid corn in five Midwestern and southern states at progressively greater distances from the eastern Iowa diffusion pole (or local origin), and also the overall pattern for the US in two spatial dimensions. Two things are immediately apparent: that lags seen when comparing arrival times in different regions would be consistent with a travelling wave contagion-diffusion explanation, and that the regional (state-level) adoption curves have different shapes and different slopes.

To incorporate this variability into our reaction-diffusion system we let the model parameters vary in space, and consider diffusion processes in spatially-varying social environments. This leads to the slightly modified equation with

$$\frac{\partial Y}{\partial t}(t,x) = D(x)\Delta Y + \xi(P(x) - P(x)Y) + \eta B(x)(1-Y)Y \,. \tag{6}$$

If we let the model parameters D, P and B vary in space we are able to model varying adoption processes in different regions. An example is shown in Figure 3. There the parameter values decrease in space with $x_1 < x_2 < \ldots < x_5$ (for simplicity we set $\xi = 0$). This constellation models the situation where regions far away from the innovation pole have smaller contact fields and weaker propensities to adopt, compared with regions closer to the innovation pole.

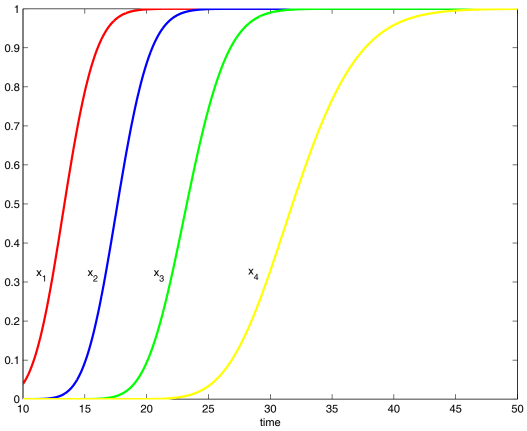


Fig. 3: Cumulative adoption curves for different regions with *D* and *B* both decreasing.

3. The Adopter Heterogeneity Framework

Contrary to the social contagion approach we focus in this section on the idea that heterogeneity in external economic factors may influence the optimal adoption timing. Individuals adopt if and only if they can afford the innovation. Such 'moving equilibrium' effects are analysed in probit models in the economics literature [6], [8], and are not accounted for in the social influence models. Based on the considerations of [25] we now illustrate the idea of the economic heterogeneity approach.

In the following we assume that the income is unequally distributed across the population, and that the price of an innovation declines as a function of time. The heterogeneity approach presumes that an individual i will adopt an innovation if its price is lower than the individual's threshold θ_i, the reservation price depending on the individual's income. We note that an unequal income distribution leads naturally to an unequal distribution of adoption price thresholds. Different considerations (e.g. [23]) have shown that it is one appropriate possibility to approximate the income of a population, and consequently the price threshold distribution, by a two parameter right-skewed gamma distribution given by the distribution function

$$f_{\Theta}(\theta) = \frac{\lambda^{\alpha}}{\alpha}\theta^{\alpha-1}e^{-\lambda\theta} \text{ with } \alpha, \lambda > 0 \text{ and } \theta \geq 0\,. \quad (7)$$

The two parameters, α and λ, can be interpreted as measures of inequality and scale. Since the determination of the individual's price thresholds is a much more difficult task compared with the determination of the individual's income, it is a common approach to model the price threshold θ by

$$\theta = cI \text{ with } 0 \leq c \leq 1$$

where I is the individual's income and c describes the propensity of spending on the innovation (cp. [26]). For a constant c the price threshold resembles the observed income distribution, which means that it is gamma-distributed and possesses the same degree of inequality (as expressed by the Gini coefficient).

Different sets of these distribution parameters lead to different degrees of inequality as shown in Figure 4 (left). There, theoretical distributions of adoption thresholds of the form (7) with the parameters $\lambda=6$ and $\alpha=2,3,4$ for a new product are presented. The levels of income inequality can be quantified by the Gini income concentration coefficient[1] of 0.375, 0.313 and 0.273. Under the assumptions of adoption threshold distributions like those shown in Figure 4 (left) and with an exponential price decline for the new product over time, the cumulative adoption curves (solid lines) in Figure 4 (right) are obtained. They are *S*-shaped, but the shape reflects income heterogeneity and not contagion-diffusion.

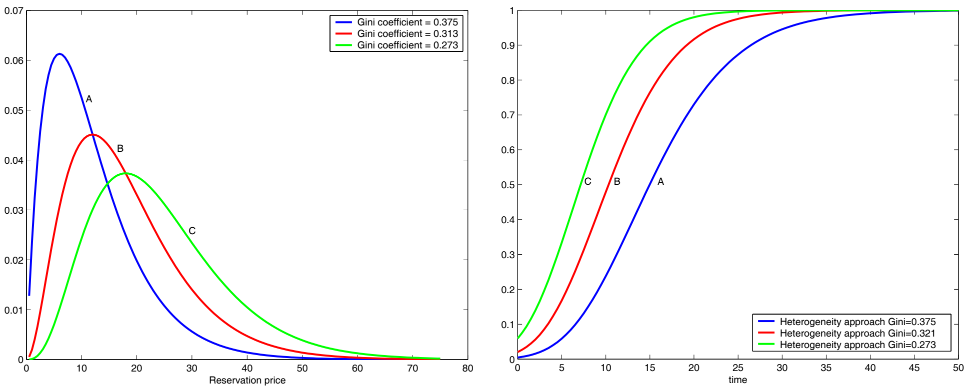


Fig. 4: Price threshold distribution for different degrees of income inequality (left) and corresponding cumulative adoption curves (right)

It is obvious that different price threshold distributions result in different adoption patterns, in a threshold heterogeneity model. A more unequal income distribution implies that a bigger proportion of the income will be earned by fewer individuals. That means that for a new product with high initial cost and a steady pattern of price decline, the diffusion of the innovation throughout the population will take longer since the price threshold of the majority of the population will be lower. Let $F(t)$ be a function which defines the proportion of the population which has adopted the innovation at time t. Assuming a gamma distribution of price thresholds for adoption, we can derive the following relations. The corresponding cumulative distribution function $F_\Theta(\theta)$ of the price threshold Θ can be interpreted as the proportion of the population with a threshold less than or equal to θ. Then,

$$F(t) = 1 - F_\Theta(\rho(t))$$

[1] The Gini coefficient is a statistical measure of the inequality of an income distribution introduced by C. Gini in [9]. It is defined as a ratio with values between 0 and 1. The closer the coefficient is to 1 the more unequal is the distribution. In the considered case of a gamma-distributed income the Gini coefficient is defined by $2B_{0.5}(\alpha\ \alpha,+1)-1$ where $B_{0.5}$ stands for the incomplete beta function [23].

where the price $\rho(t)$ of the innovation is a decreasing function in time. As a further consideration we assume an exponential price decline of the form

$$\rho(t) = \rho_0 e^{-bt}$$

(where the innovation has an initial price of ρ_0). In this case we can determine the change $\partial F(t) / \partial t$ in the proportion which has already adopted at time t by

$$\frac{\partial F(t)}{\partial t} = f(t) = -F_{\Theta}^{'}(\rho(t))\rho^{'}(t) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \rho_0^{\alpha} b e^{-\alpha bt - \lambda e^{-bt}} .$$

4. Combining the Effects of Social Influence and of Economic Heterogeneity

As shown in the previous sections both approaches - based on the social influence hypothesis and on the economic heterogeneity hypothesis - provide good fits for *S*-shaped cumulative adoption curves. The social influence model assumes that the spread behaviour is determined by independent assessment and by copying processes in the population. This approach assumes implicitly that every individual who wants to adopt is able to do so, and that there are no external economic constraints on the adoption decision. In contrast, the economic heterogeneity model is based on such an external constraint, and assumes that the adoption process is determined only by the individual's reservation price threshold - the ability to afford the innovation. However, in this case it is assumed implicitly that all individuals in the population are aware of the innovation and want to adopt it as soon as possible.

It might seem more appropriate to combine both hypotheses and to consider social influence and affordability simultaneously. We therefore now develop a model in which the desire to adopt is influenced by density-dependent social influence processes, but the timing of adoption is constrained by affordability (a detailed analysis of this model can be found in [18]). This leads to the following approach

$$X(t) = Y(t)F(t) = Y(t)(1 - F_{\Theta}(\rho(t))) \tag{8}$$

where F is the cumulative adoption curve obtained by the heterogeneity approach and Y the cumulative adoption curve of the social influence model given by equation (4). This model assumes that acquiring a preference to adopt the innovation is independent of income. People are heterogeneous with respect to price thresholds for adoption, but homogeneous with respect to mechanisms for acquiring the preference to adopt.

We have seen that economic inequality can slow the spread of the innovation. We now analyse the spread dynamic of the combined model and consider the influences of different patterns of price decline. At first we assume that the price is constant over the whole time period. Figure 5 (left) illustrates this situation with a constant price ρ=10, an unequal income distribution indexed by a Gini coefficient of 0.375 and Bass parameters p=0.0035 and q=0.15. The cumulative adoption curve of the heterogeneity approach is simply a constant, since the price does not change from the initial value. The cumulative adoption curve of the combined model shows an *S*-shaped pattern. It is dominated in the first time period by the density-dependent copying process, but economic constraints prevent the spread of the innovation through the whole population and cause a cut-off point (because half the population are unable to afford the innovation, even though social influences may cause them to want to do so).

In contrast in Figure 5 (right) we assume that the price decreases exponentially; all other parameters are the same as in the preceding example. The price decline has the form $p(t)=100e^{-0.03t}$ (a high initial price and a slow price decline). In this case the 'willingness' to adopt the innovation spreads faster than its affordability and the shape of the cumulative adoption curve of the combined model is dominated by the affordability constraints. At the time where $Y(t)=1$ yields the curve converges with the cumulative adoption curve from the pure economic heterogeneity approach.

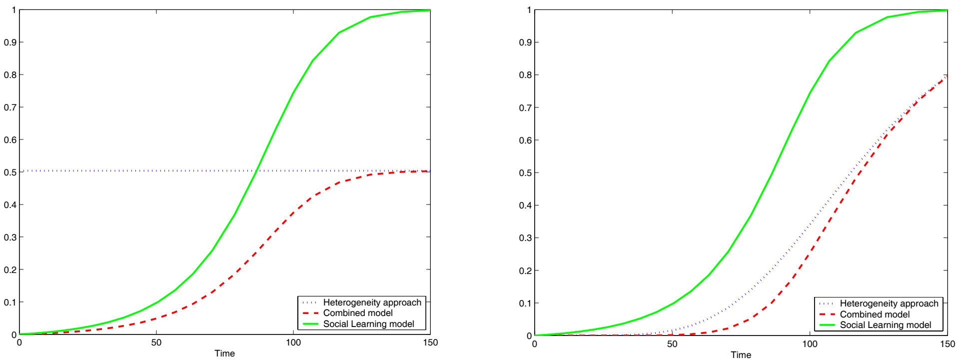
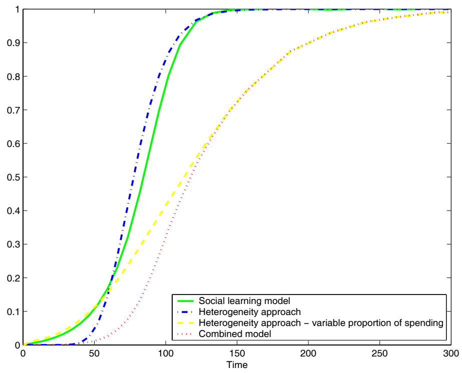


Fig. 5: Comparison of the cumulative adoption curves for a constant price *p* = 10 (left) and for an exponential price decline *p*(*t*) = 100e¡0:03t (right), income inequality given by the Gini coefficient of 0.375 and Bass parameters *p* = 0:0035, *q* = 0:15

These examples demonstrate clearly that economic factors have to be included, where they would counteract or delay the spread predicted when only learning mechanisms are considered.

To further increase the realism of the model we now want to relax the implicit assumption of the heterogeneity approach mentioned above, namely that all individuals will have the same propensity of spending a constant fraction of their income on an innovation. This means that we now incorporate the fact that the ratio between discretionary income and income is higher for wealthier individuals and tends to zero for individuals with a very small income. Figure 6 illustrates the approach. The dashed line shows the previous assumption of a constant propensity of spending regardless of income level. Now we assume a quadratic dependence between the income and the propensity of spending (solid line). This leads to a higher number of adopters at the beginning but also decreases the proportion of the population which will adopt at late time points, compared with the situation of a constant propensity. Figure 7 makes that effect obvious. The dotted line represents the cumulative adoption curve for a heterogeneity approach with a constant propensity of spending, whereas the chained line shows the cumulative adoption curve produced by the income dependent propensity given in Figure 6 (solid line). At the time period from t=0 to roughly t=50 the adoption curve of the 'variable' heterogeneity approach (dashed line) is above the curve of the 'constant' heterogeneity approach (dotted line) since the wealthier individuals tend to spend a higher proportion of their income on the innovation. Then as time goes by, the price of the innovation decreases so that more individuals can afford the innovation. But the ‘variable’ heterogeneity approach assumes that the poorer the individuals, the smaller the proportion of their income they spend on the innovation. Therefore the chained line is below the dotted line for later times. This causes a long tail at the end. If we combine now the social influence model with the ‘variable’ threshold heterogeneity approach we obtain a cumulative adoption curve (dashed line) showing an *S*-shaped pattern with a longer tail at the beginning (caused by the social influence processes) and at the end (caused by the proportionally-varying distribution of reservation prices).



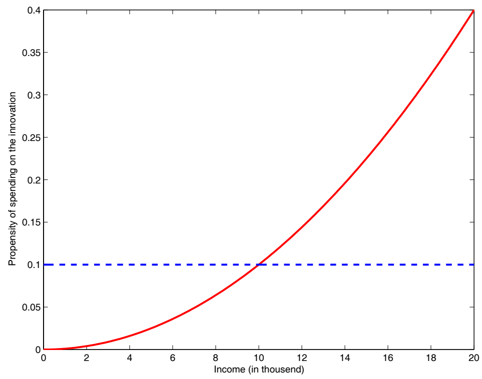


Fig. 6: Proportion of the income which is spent on the innovation

Fig. 7: Cumulative adoption curves of the different approaches

4.1. The case of black and white TV adoption in the US

We now discuss in more depth a concrete example. In the case of the adoption of black and white TV in the US, Wang has argued in [26] that the observed delay reflects income inequality. When the new product was introduced in 1946, high-income consumers tended to adopt it first. The price then fell with cumulative output, and demand grew as the product penetrated into lower income groups. Based on the considerations in [2] for the time period from 1946 to 1960 we assume an exponential price decline for black and white TVs of the form

$$\rho(t) = 1283e^{-0.087t} .$$

This implies that the initial price of black and white TV at the market launch was 1,283, and that afterwards a relative steep price decline was observable. We adjust this to take account of the simultaneous growth in nominal per capita GDP in the USA, which we take as a proxy for average nominal per capita income. Based on data for the period 1946-1971 in [16] we estimate this exponential rate of economic growth as $e^{0.045}$, which means that the price decline for black and white TVs (as a fraction of average income) can be approximated as

$$\rho(t) = 1283e^{-0.1322t} .$$

The family income distribution is approximated by a gamma distribution and we use the parameter α=2.49 and $\lambda=3.9\cdot10^{-4}$ estimated in [20] for the year 1960. The corresponding Gini coefficient is 0.34, which shows that family income in 1960 was relatively unequal distributed. Evidence suggests that the Gini coefficient was fairly stable and constant during the period of diffusion of this innovation.

Bass [1] estimated his model from TV sales data and obtained the coefficient of innovation p as 0.0279 and the coefficient of imitation q as 0.25. Bayus ([2]) also fitted the Bass model to the actual sales data but estimated the coefficient of innovation p by 0.0159 and the coefficient of imitation q by 0.39. We now attempt to model the diffusion rate for black and white TVs using the threshold heterogeneity model, but using these two alternative Bass curve fits as our targets for comparison. We note in passing that in contrast with the threshold-based curves, these best-fit Bass curves are not constrained by any independent empirical data on the strength of imitative bias among adopters.

In Figure 8 (left) the dotted line represents the curve obtained by the threshold heterogeneity approach with c=1/11 as constant propensity of spending on black and white TV (which means the individuals or families would spend up to about 10 percent of their annual income on their first black and white TV). The comparison with the two fitted Bass curves shows that the general behaviour is similar although it does not fit exactly.

In contrast, the dashed line represents the cumulative adoption curve obtained by the combined model with the income distribution of the year 1960 for the heterogeneity approach and Bass parameters p=0.06, q=0.3. This means that compared with the pure Bass model as estimated by Bass or by Bayus, the probability is slightly increased that an individual will decide to adopt the innovation independently of social influence. Figure 8 (right) differs in that we now assume a variable proportion of income as discretionary in the combined model, which now has Bass parameters p=0.035, q=0.35. It is evident that the threshold distribution predicts the correct timescale for the diffusion process, and that the social contagion component of the combined model improves the fit to the Bass curves by delaying take-off in the higher-income groups.

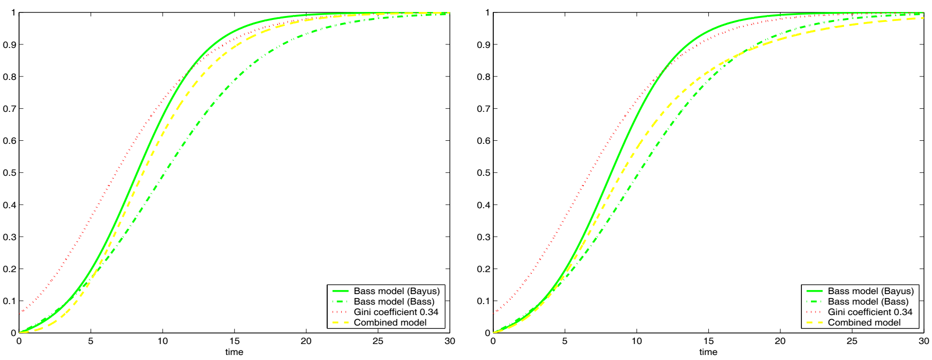


Fig. 8: Cumulative black and white TV adoption curves for the Bass, threshold and combined models. The Bass curves were fitted to actual sales data, while the threshold curves are based on income distribution and on the exogenous growth-adjusted price trend. Threshold set as a constant fraction (left) and a variable fraction (right) of income.

Summarizing, our empirical threshold distribution gives a good fit to the Bass sales curves for this case study, and the combined model improves that fit by introducing a density-dependent copying bias to explain the observed delay in take-off. Overall, we infer that inequality in the population's income distribution is indeed likely to be a very important factor in explaining the time course of new product diffusion.

4.2. Spatial combined model

We have seen that an approach combining social influences and external factors as affordability may be able to explain the time delay in the adoption decision within a given region accurately. In this section we generalise the combined model (8) by introducing spacial dependency. In the light of Hägerstrand's framework we develop a spatially explicit model which considers both, assessment of the innovation's efficacy, and external constraints on affordability.

Our approach is based on equation (6) which describes the adoption dynamic within a spatially structured population under the assumption that every individual who wants to adopt can do so (The likelihood of the adoption depends only on the adoption parameters *P* and *B*, and the number of adopters in the neighbourhood.) Contrary, we assume now the adoption decision is constrained by affordability and combine the reaction-diffusion framework with the heterogeneity framework. We obtain the following approach:

$$\frac{\partial X}{\partial t}(t,x) = D(x)\Delta X + \xi(P(x)F - P(x)X) + \eta B(x)\left(1 - \frac{X}{F}\right)X + \frac{\partial F}{\partial t}\frac{X}{F}. \quad (9)$$

Analogously to the previous consideration the space- and time-dependent function X models the cumulative adoption curve for different regions x and the time-dependent function F models the proportion of the population who can afford the innovation at time t. So the shape of the adoption curves is determined by the spread of the information of the innovation through the population and affordability. Thereby we may also consider the function F as space- and time-dependent to account for regional differences in the income structure. In this case the diffusion term $D(x)\Delta X$ is replaced by

$$\frac{D(x)}{F^2}\left[\sum_{i=1}^{2}\left(\left(\frac{\partial^2 X}{\partial x_i^2}F - \frac{\partial^2 F}{\partial x_i^2}X\right)F - 2\left(\frac{\partial X}{\partial x_i}F - \frac{\partial F}{\partial x_i}X\right)\frac{\partial F}{\partial x_i}\right)\right]$$

with $x=(x_1,x_2)$. Summarising, model (9) describes the spatiotemporal adoption pattern of an innovation which is influenced by social transmission mechanisms and external factors such as affordability.

We now briefly describe two empirical cases of innovation diffusion, which highlight the need of considering different diffusion hypothesis and temporal as well as spatial dependency. In the next phase of our work we intend to fit values for the parameters of the alternative diffusion models.

In the first case, we illustrate the large-scale spatiotemporal diffusion of a higher-yielding hybrid strain of corn which was introduced into US agriculture in the middle of the last century (Figures 2 and 9). The adoption pattern shows many of the features we have been discussing, including regional variation (at the state level) in the shape of the cumulative adoption curve (Figure 2). In this case, while a contagion model could be fitted using the above social environmental parameters, a threshold heterogeneity explanation of the observed lag would need to refer to locally-varying agricultural factors.

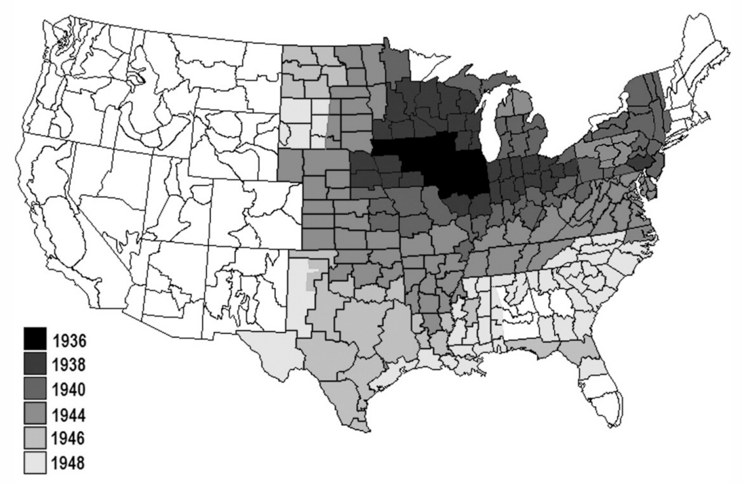


Fig. 9: Diffusion of hybrid corn usage, showing areas that planted 10 or more percent of their corn acreage to hybrid seed at successive time intervals (redrawn after [12], Figure 3).

Griliches [10], [12] preferred an explanation in terms of unequal supplier effort, with commercial seedcorn suppliers initially targeting regions with large farm units and high corn acreage (and contiguous areas with similar climate, soil and pest characteristics). However, commercially-acquired hybrid seedcorn also cost almost ten times as much to adopters as home-grown seedcorn [11], and the extra yield also imposed potential additional input costs (supplies of fertiliser and water, and extra labour for harvesting). The extra cost of hybrid seed represented a fixed cost per unit area, whereas yield varied [7]; and it is plausible therefore that late adopters included farms with lower typical corn yields, for whom the high relative cost of the new strain was a significant factor delaying the adoption decision. The same point is made by David [6], by analogy with his analysis of threshold heterogeneity effects on adoption timing for the mechanized reaper in the antebellum American MidWest [5].

In the second case, we illustrate the pattern of diffusion of tractors in Illinois in the same period (Figure 10). In this case, the time series of maps illustrates a spatiotemporal lag, but does not in itself enable us to diagnose local spatial variation in the parameters of the social influence model.

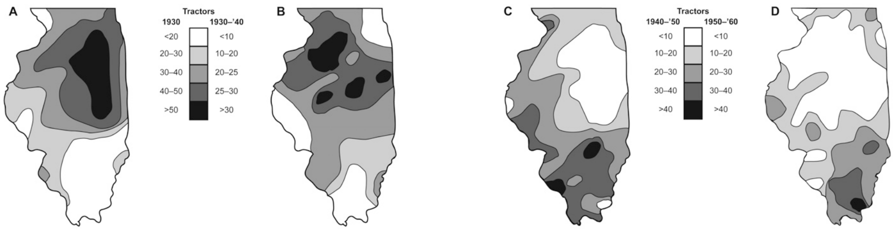


Fig. 10: The diffusion of the tractor in Illinois (redrawn from [19]).The first (1930) map shows the percentage of farms with tractors; the subsequent maps show the difference in percentages between the beginning and end of the indicated time periods.

A good fit for a threshold heterogeneity model is however implied by the parallel between the observed lags, and spatial variation in average farm sizes and crop regimes (Figure 11).

We now intend to fit empirical values for the parameters of the two models in both cases, as a further extension of our analysis.

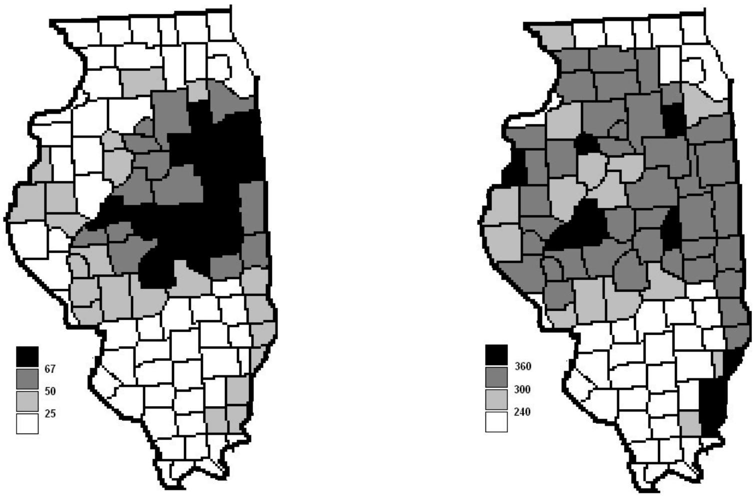


Fig. 11: (left) 1949 data on percentage of farms focused on cash-grain cropping, by county (redrawn after [14], Figure 13) (right) Average farm size in acres, by county, 1982 (redrawn after [14], Figure 5). Overall in the twentieth century farm size has increased, but the county-by-county pattern of relative sizes for Illinois is conserved: 1939 farm sizes predict 1982 farm sizes with high accuracy (r = 0.94, [14]).

5. Conclusion

In this paper we considered the potential effects of social contagion processes and of economic inequality on innovation diffusion rates in time and space and showed that the typical evolution of the distribution of adopters may be consistent with either explanation. We have then attempted to integrate the insights of the two approaches in a single model that incorporates both social contagion and threshold heterogeneity effects.We have introduced a spatial reaction-diffusion extension of a social contagion approach and the 'combined' model, and we have shown that a spatial dependence for economic inequality can produce spatiotemporal diffusion patterns very similar to those produced by a spatially-dependent social contagion process. We have illustrated our analysis with well-documented examples from modern market economies; extending our analysis to historical cases and to cases from pre-industrial societies may require data with exceptionally high temporal and spatial resolution.

Our findings have implications for the experimental analysis of consumer choice behaviour, as well as for optimal pricing strategy for new products. More generally, they confirm that empirical patterns of temporal and spatio-temporal innovation diffusion will not usually indicate the dominance of one or other factor in adoption decisions. Such a diagnosis will require additional consideration of the distribution of a range of social and economic variables.

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