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How to Measure Subdiffusion Coefficient

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Abstract

We propose a method to measure a subdiffusion coefficient D_α. The method, which exploits a membrane system, relies on the so-called near-membrane layers. We experimentally study the diffusion of glucose and sucrose in a gel solvent. We find a fully analytic solution of the fractional subdiffusion equation with the initial and boundary conditions representing the system under study. Confronting the experimental data with theoretical results, we find the values of the subdiffusion coefficient for investigated substances.

Keywords: subdiffusion.

1. Introduction

The subdiffusion is characterized by the relation

$$\left\langle x^2 \right\rangle = 2D_\alpha t^\alpha / \Gamma(1+\alpha),\ \ \alpha < 1 \qquad (1)$$

where $\left\langle x^2 \right\rangle$ denotes the mean-square displacement of the particle which starts from $x=0$ at the initial time $t=0$. The case of $\alpha=1$ corresponds to the normal diffusion. For practical reason, we choose for the experimental study a membrane system containing two vessels with a thin membrane in between which separates the initially homogeneous solute of the substance of interest from the pure solvent. Instead of the mean-square displacement (1), our method refers to the temporal evolution of the thickness δ of the near-membrane layers (NML). The NML is defined as the distance from the membrane where the substance concentration drops κ times with respect to the membrane surface (κ is an arbitrary number). We show that for subdiffusion [1,2]

$$\delta(t) = A(D_\alpha, \kappa, \alpha)\, t^{\alpha/2}. \qquad (2)$$

Comparing the prediction of eq. (2) with experimental data, we find the values of the coefficients α and D_α.

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2. The method

The substance concentration is measured by means of the laser interferometric method. When the sugar was diffusing across the membrane, we were recording the concentration profiles in the vessel which initially contained pure gel (1.5% water solution of agarose) [1,2]. The thickness of a near-membrane layer was calculated from the measured concentration profiles *C(x,t)* according to the definition

$$C(\delta,t)=\kappa C(0,t), \qquad (3)$$

where $x=0$ is the location of the membrane.

The subdiffusion is described by the equation with Riemann-Liouville fractional derivative

$$\frac{\partial C(x,t)}{\partial t}=D_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}}\frac{\partial^2 C(x,t)}{\partial x^2}\ .$$

Solving the equation with an appropriate initial condition ($C(x,0)=C_0$ for $x<0$ and $C(x,0)=0$ for $x>0$) and two boundary conditions at the membrane (the first one assumes the continuity of the flux flowing through the membrane, the second involves a linear combination of flux and concentrations) we find that in the long-time approximation the concentration of sugar for $x>0$ is [1-3]

$$C(x,t)=B\cdot H_{11}^{10}\left(\left(\frac{x^2}{D_\alpha t^\alpha}\right)^\alpha \middle| \begin{matrix} 1 & 1 \\ 0 & 2/\alpha \end{matrix}\right), \qquad (4)$$

where *H* denotes Fox function, *B* is a coefficient which is not important for the further considerations. From the equations (2)-(4) we obtain

$$D_\alpha = A^2 \Big/ \left[\left(H_{11}^{10}\right)^{-1}\left(\frac{\alpha\kappa}{2}\middle| \begin{matrix} 1 & 1 \\ 0 & 2/\alpha \end{matrix}\right)\right]^\alpha .$$

Knowing experimental values of A, α, and κ, we can deduce D_α.

3. Final remarks

The experimental value of subdiffusion parameter $\alpha=0.90\pm0.01$ is the same for glucose and sucrose. The subdiffusion coefficient D_α equals $D_\alpha=(9.8\pm1.0)\mathrm{x}10^{-4}\mathrm{mm}^2/\mathrm{s}^{0.90}$ for glucose, and $D_\alpha=(6.3\pm0.9)\mathrm{x}10^{-4}\mathrm{mm}^2/\mathrm{s}^{0.90}$ for sucrose. The time evolution of the near-membrane layer appears to be fully independent of boundary conditions at a membrane. Therefore, it is expected that the membrane properties do not influence the time evolution of the near-membrane layer.

References

[1] T. Kosztolowicz, K. Dworecki, S. Mrowczynski, Phys. Rev. Lett. 94 (2005) 170602

[2] T. Kosztolowicz, K. Dworecki, S. Mrowczynski, Phys. Rev. E 71 (2005) 041105

[3] T. Kosztolowicz, J. Phys. A 37 (2004), 10779.