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Dynamics of tagged particles in a biased $A + A \rightarrow \Phi$ system in one dimension: result for asynchronous and paallel updates

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We have studied the dynamical features of tagged particles in one dimensional $A + A \rightarrow \Phi$ system, where the particles A have a bias ε $(-0.5 \le \varepsilon \le 0.5)$ such that they move towards their nearest neighbour with the probability 0.5+ε and move in the opposite direction with probability 0.5-ε and the particles are annihilated on contact. ε=0 implies purely diffusive motion and ε=0 represents purely deterministic motion of the particles. The principal point of interest of studying these systems lies in the manner in which the system approaches the steady state and the concentration of particle changes with time.

The fraction of walkers $\rho(t)$ at time t was found to decay as $\rho(t) \sim t^{-\alpha}$, where α=1/2 in the absence of the bias [1, 2]. If asynchronous dynamics is used, it is known that $\alpha=1$, when the bias, however small, is introduced [3]. Though the walkers, in the long time limit, seemed to behave as ballistic walkers, several features (e.g. persistence, domain growth) show that it is actually not the same. Hence, to get a better understanding we study the dynamics of a tracer walker in the biased case specifically to check whether they perform ballistic motion or not. The idea behind all these studies is to find any universal behaviour in these systems and the key factors which determine the universality. We have used two different dynamics to update the system; asynchronous and parallel. In asynchronous dynamics particles are updated randomly and one after another. In parallel dynamics position of the particles are updated together; the annihilation takes place after the completion of one Monte Carlo step.

We found that for ε>0 when asynchronous dynamics is used to update the system, the probability distribution $\pi(x, t)$ that the particle has undergone a displacement x at time t shows a double peak structure with a dip at $x=0$ and it assumes a double delta form at very late time regime. For any ε>0, there is a time scale $t^* \sim (\varepsilon_c-\varepsilon)^{-1}$, which diverges at ε_c=0.5, the critical point of the dynamics. We have detected a crossover behaviour from the annihilation dominated to a diffusion dominated regime at time t^*. Below t^*, motions are highly correlated when the particle density is large and number of annihilation is considerable and beyond t^*, the particles move as independent biased walkers. When we use parallel updating rule for ε>0, $\pi(x, t)$ shows a non-Gaussian single peaked structure and $\rho(t)$ shows $\ln t/t$ variation. We found that an isolated pair of particles, termed as dimers, can survive indefinitely in the system for ε=0.5, which is exclusive for parallel dynamics.

We also found that the persistence exponent in the parallel case seems to be twice of the one found in the asynchronous case for ε=0. Such doubling of persistence exponent could be proved for the Ising Glauber or Potts model with parallel dynamics. Although for asynchronous dynamics, the $A + A \rightarrow \Phi$ model with ε=0 and the Ising Glauber model are identical, with parallel dynamics, such a correspondence no longer exists. For negative values of the bias ε, the particles are biased to move away from their nearest neighbour, $\pi(x, t)$ becomes Gaussian for both the dynamics.

A comparative analysis for the behaviour of all the relevant quantities for the system using asynchronous and parallel dynamics shows that there are significant differences for ε>0 while the results are qualitatively similar for ε<0. The present study is able to manifest at the individual level the precise role of the bias and how the dynamics are different from simple ballistic motion.

References

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